

### **Derivative of the Square Root Function**

- a) Use implicit differentiation to find the derivative of the inverse of  $f(x) = x^2$  for  $x > 0$ .
- b) Check your work by finding the inverse explicitly and then taking its derivative.

$$a) f(x) = x^2, \quad x > 0$$

$$\text{Let } y = f(x) \Rightarrow y = x^2 \Rightarrow \sqrt{y} = x$$

$$f^{-1}(y) = x, \quad f^{-1}(y) = \sqrt{y}$$

$$\Rightarrow \frac{d}{dx} f^{-1}(y) = \frac{d}{dx} x$$

$$\frac{d}{dy} f^{-1}(y) \frac{dy}{dx} = 1$$

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{2x} = \frac{1}{2\sqrt{y}}$$

$$\Rightarrow \frac{d}{dx} f^{-1}(x) = \frac{1}{2\sqrt{x}}$$

$$b) f(x) = x^2$$

$$f(f^{-1}(x)) = (f^{-1}(x))^2$$

$$x = (f^{-1}(x))^2$$

$$f^{-1}(x) = \sqrt{x}$$

$$\therefore \frac{d}{dx} f^{-1}(x) = \frac{1}{2} x^{-1/2}$$

$$= \frac{1}{2\sqrt{x}}$$